Derivation of the Relationship Between B^* and $\dot{n}_0$ / 2

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Background

During 1986-87, my experience with the NORAD "2-line" orbital elements and the SGP and SGP4 orbit propagators, revealed that as new element sets were issued, SGP4's B^* decay element varied in direct proportion to SGP's $\dot{n}_0$/ 2 decay element. This knowledge proved useful in updating the decay elements of objects such as the Mir space station, based on observations. It was easy to estimate new values of $\dot{n}_0$/ 2, based on the difference between the predicted and observed time of passage of a satellite, which in turn was used to update B^*, in direct proportion to the change in $\dot{n}_0$/ 2.

In 1988, I began to receive "2-line" orbital elements derived by non-NORAD sources, which were intended for use only in SGP, and thus omitted the B^* element. This was inconvenient for myself and others who had migrated from SGP to SGP4, so having observed that B^* varied in direct proportion to $\dot{n}_0$/ 2, I decided to attempt to derive the mathematical relationship between them.

Derivation

SGP propagates semi-major axis, a, by:

$$a = a_0 \left(\frac{n_0}{n_0 + 2\left(\frac{\dot{n}_0}{2}\right)\Delta t + 3\left(\frac{\ddot{n}_0}{6}\right)\Delta t^2} \right)^{2/3}$$

where:

- a_0 = semi-major axis at epoch, as computed during SGP initialization.
- n_0 = mean motion at epoch, an SGP element.
- $\dot{n}_0$/ 2 = 1/2 of time rate of change of mean motion at epoch, an SGP element.
- $\ddot{n}_0$/ 6 = 1/6 of second time rate of change of mean motion at epoch, an SGP element.
- Δt = time since epoch.

SGP4 propagates semi-major axis, a, by:

$$a = a_0''\left(1 - C_2 B^* \Delta t - D_2 \Delta t^2 - D_3 \Delta t^3 - D_4 \Delta t^4\right)^2$$

where:

- a_0'' = semi-major axis at epoch, as computed during SGP4 initialization.
- B^* = SGP4 decay element.
- C_2, D_2, D_3, D_4 = constants computed during SGP4 initialization. D_n are functions of B^*.
- Δt = time since epoch.

To simplify the solution, all but the first order Δt terms were dropped from both equations. I have never seen the $\ddot{n}_0$/ 6 element used by non-NORAD sources that did not also include B^*. SGP4's higher order Δt terms contribute little to the overall change in semi-major axis. For example, in propagating space station Mir several months into the future, they contribute less than 0.1 percent to the change in semi-major axis. Also, SGP4 drops the higher order terms for perigee heights below 220 km; and its deep-space version, SDP4, does not use them at all.

Dropping all but the first order Δt terms, and solving both equations for Δt, leads to the following relationship between B^* and $\dot{n}_0/2$:

$$B^* = \frac{2\left(\frac{\dot{n}_0}{2}\right)}{C_2 n_0} \bullet \frac{1-\left(\frac{a}{a_0''}\right)^{1/2}}{\left(\frac{a}{a_0}\right)^{-3/2} - 1}$$

At the epoch, a equals a_0 and a_0'', leading to the following simplification:

$$B^* = \lim_{\substack{a \to a_0'' \\ a \to a_0}} \frac{2\left(\frac{\dot{n}_0}{2}\right)}{C_2 n_0} \bullet \frac{1-\left(\frac{a}{a_0''}\right)^{1/2}}{\left(\frac{a}{a_0}\right)^{-3/2} - 1}$$

$$B^* = \frac{2\left(\frac{\dot{n}_0}{2}\right)}{3 C_2 n_0}$$

The accuracy of this equation has been confirmed by a test on a large subset of the NORAD catalogue, consisting of 8,093 elements near epoch 99293. Elements were judged suitable for comparison if they were of NORAD origin, B^* and $\dot{n}_0/2$ had the same sign, and neither were obvious fixed values. NORAD uses fixed decay rates for thousands of low-drag objects.

The following Table summarizes the difference between NORAD's B^* and the estimated value, for the 4,844 elements suitable for comparison:

NORAD's B^* vs. Estimated B^*

Difference: ± %	Count	% of Total
1	1569	32.4
2	2219	45.8
5	3055	63.1
10	3683	76.0
25	4404	90.9

Nearly two-thirds of the B^* differences were within ± 5 percent. Fortunately, the largest percentage differences occurred at the *lowest* $\dot{n}_0/2$, increasing asymptotically for values of $\dot{n}_0/2$ < 0.00002. More than 95 percent of the 2,349 elements with $\dot{n}_0/2$ > 0.00002, had B^* differences within ± 5 percent. I have used the estimated B^* successfully for ten years.